SOME EXPERIMENTS TESTING THE MOST RAPID METHOD OF FACTORING THE TYPE $ax^2 + bx + c$.

By Fiske Allen.

For some time there has been considerable debate among the mathematics teachers of Horace Mann High School as to the best method of factoring the quadratic trinomial of the type $ax^2 + bx + c$. It was rather generally agreed that the so-called "cross-product" method is most easily made rational to the pupil and that the method of "splitting the middle term" is least easily taught; also that guessing is involved in all the methods. But it is insisted that either the "splitting-middle-term" method or "multiply-by-a" method is more scientific than the "cross-product" method because a definite system of guessing can be given for the first two which does not apply to the third. This last argument is denied, with no possibility of proof or conviction, but a disagreement upon the question of which method is most rapid was more easily tested and the following experiments made.

We had three sections of our first-year class taught by three different teachers and all were ready to begin studying this type of factoring on the same day. We assigned to class A the method of "splitting-middle-term," to class B the "cross-product" method and to class C the "multiplying by a" method. Instruction in the three classes was to begin on the same day and drill was to continue till the teachers agreed they were ready for a test. We agreed to select for the test the most difficult list on this type we could find in any text and the list of twenty-four problems found in Milne's "College Algebra" was selected. The pupils were told to solve as many as possible in exactly thirty minutes. The results of this test showed class A had solved an average of 8 problems, class B 15, class C 6.

Next, in order to test the relative abilities of the classes and also of the teachers, it was decided to hold two more tests on factoring types taught exactly alike to the three classes. For

this purpose we first selected the type $a^2 - b^2$, drilled on it the same length of time and on the same day gave the same test to the three classes. The results of this test showed class A 21.8, class B 15.3, class C 16. Our next ability test was on the types $a^3 \pm b^3$ and $a^5 \pm b^5$. This test showed class A 17.2, B 14, C 15.2.

These results were so conclusive that the opponents of the cross-product method set to work to find the weak points in the experiments. The most vulnerable point was the fact that the teacher of class A was not enthusiastic for the method he was teaching but preferred the cross-product. And the point was also made that the problems chosen were too easy and generally "first-guess" problems. It was asserted that the cross-product method would fail of the rapidity of the other methods if larger numbers with more possible factors were given. To obviate the first of these objections we induced a former teacher in Horace Mann and an enthusiastic advocate of method A to teach class A for four days. He was to have one day to become acquainted with the class, the other teachers giving but three days to drill. At the close of that period another test was given. The problems were made to order, since none of sufficient difficulty could be found in any text and all first guess problems were eliminated. The list was submitted to each teacher and any problem peculiarly adapted to another teacher's method was eliminated. Because of the difficulty of the problems the results of this test were very low, as follows: Class A 4.7, B 6, C 3.5.

This test was given the last day preceding the Easter vacation. Immediately following it two more tests were given on the same list of problems with results as follows: First test—A 3, B 5.6, C 2.8; second test—A 3.7, B 5, C 2.9. It was then decided to give two more ability tests, the first on the type $x^4 + x^2y^2 + y^4$. In this test class A solved 15, B 13, C 18.4. Of the type factored by the "Factor Theorem" A solved 3.5, B 4.8, C 2.1.

Averaging the results of the various tests we find that on the type $ax^2 + bx + c$ class A solved 4.9, B 7.9, C 3.8. On the ability tests the average was class A 14.4, B 11.8, C 12.9. Considering the relative ability of the three classes as shown by the ability tests the results of the $ax^2 + bx + c$ type *should have*

been: A 9.6, B 7.9, C 8.5. Comparing the *actual* results with those which should have been shown had all classes been taught the cross-product method we find the method of "splitting the middle term" barely half as efficient as the other, and "multiplying by a" little more than one third as efficient.

We do not regard this single experiment as at all conclusive, but it points so emphatically to one conclusion that we shall be much interested in having our results corrected or corroborated by similar experiments others may make.

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Purge out of every heart the lurking grudge. Give us grace and strength to forbear and to persevere. Offenders, give us the grace to accept and to forgive offenders. Forgetful ourselves, help us to bear cheerfully the forgetfulness of others. Give us courage and gaiety and the quiet mind. Spare to us our friends, soften to us our enemies. Bless us, if it may be, in all our innocent endeavours. If it may not, give us the strength to encounter that which is to come, that we be brave in peril, constant in tribulation, temperate in wrath, and in all changes of fortune, and down to the gates of death, loyal and loving one to another.—*Robert Louis Stevenson.*